RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. SIXTH SEMESTER EXAMINATION, MAY 2015

THIRD YEAR

Date : 30/05/2015 **MATHEMATICS (Honours)**

Time : 11 am – 2 pm **Paper :** VIII Full Marks : 70

Group – A

[Use a separate Answer book for each Unit]

Unit – I

[*Answer **any five** questions*]

1. If a system of Coplanar forces acting at different points of a rigid body have a single resultant and if each force be turned in the plane of the forces about its point of application through the same angle in the same sense, prove that their resultant will always pass through a fixed point. [6]

2. A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ, ϕ be the inclinations of the string and the plane base of the hemisphere to the vertical, prove that $\tan\phi = \frac{3}{8} + \tan\theta$. [6]

3. A, B are two points on the same level at a distance 'a'. A uniform rod AC of length 'a' and weight $2\sqrt{3}\,\text{lb}$. is hinged at A and is supported by a string fastened at C, passing over a smooth pulley at B and carrying a weight of one pound. Find the equilibrium position and show that it is stable. [6]

4. Deduce the conditions of equilibrium of a system of forces acting on a rigid body from the principle of virtual work. [6]

5. Three forces each equal to Q, act on a body, one at the point (1,0,0) parallel to Oy, the second at the point (0,1,0) parallel to Oz, the third at the point (0,0,1) parallel to Ox; the axes being rectangular, find the equation of the central axis. [6]

6. Two equal uniform ladders are joined at one end and stand with other ends on a rough horizontal plane. A man whose weight is equal to that of a ladder ascends on one of them. Prove that the other ladder will slip first. If it begins to slip when he has ascended a distance 'd', prove that the coefficient of friction is $\left(\frac{\ell+d}{2\ell+d}\right)\tan\alpha$, '$\ell$' being the length of each ladder and 'α' the angle each makes with the vertical. [6]

7. Determine the Cartesian equation of equilibrium of a string, when the string is not uniform, and m, the mass per unit length of the string at any point, is proportional to the tension at that point, i.e $m \propto T$ and gravity is the only external force. [6]

8. A uniform string rests in limiting equilibrium on a rough vertical circle and subtends an angle β at the centre. If the string is on the point of slipping off, prove that the angular distance α of its upper end from the highest point of the circle is determined by the equation, $\cos(\alpha+\beta-2\lambda) = e^{\beta\tan\lambda}.\cos(\alpha-2\lambda)$, where λ is the angle of friction. [6]

Unit – II

[*Answer **any two** questions*]

9. a) State the Duality Principle. Express the Boolean expression $(xy + x'z)'$ in CNF and DNF in x,y,z. [5]

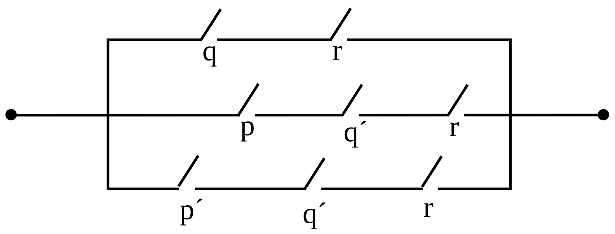
b) Write a C-program to determine the repeated fraction

$$f(20)+\cfrac{1}{f(19)+\cfrac{1}{\dots+\cfrac{1}{f(2)+\cfrac{1}{f(1)}}}}$$

where $f(n)=n^3+2^n$, using function. [5]

10. a) Write a C-program to input two matrices of order m×n and p×q respectively and find their product when they are conformable for matrix multiplication. [5]

b) Simplify the following switching circuit : [5]



11. a) Write a C-program to find the standard deviation (σ) of N values of a variable X, $x_1, x_2, \dots, x_N$ defined by $\sigma=\sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2}$ where $\bar{x}=\frac{1}{N}\sum_{i=1}^{N}x_i$ is the A.M of the N items. [4]

b) Write short notes on : BASIC, JAVA. [4]

c) State few differences between compiler and interpreter. [2]

(Answer either Unit-III or Unit – IV)

Unit – III

[*Answer **any two** questions*]

12. a) If A_i is a covariant vector show that $\frac{\partial A_i}{\partial x^j}-\frac{\partial A_j}{\partial x^i}$ are the components of a tensor. [3]

b) The components of a covariant tensor of rank 2 in the co-ordinate system x^i are $A_{11}=2$, $A_{12}=A_{21}=8$, $A_{22}=5$. Find its components in the co-ordinate system $\bar{x}^i$, where $x^1=2\bar{x}^1-3(\bar{x}^2)^2$, $x^2=4\bar{x}^1-\bar{x}^2$. [4]

c) Show that the vector $\left(-1,0,0,\frac{1}{c}\right)$ is a null vector for the space whose metric is $(ds)^2=-(dx)^2-(dy)^2-(dz)^2+c^2(dt)^2$. [3]

13. a) Deduce that $\frac{\partial g^{ik}}{\partial x^j}=-g^{hk}\begin{Bmatrix} i \\ hj \end{Bmatrix}-g^{hi}\begin{Bmatrix} k \\ hj \end{Bmatrix}$, where the symbols have their usual meaning. [4]

b) Show that the coefficients g_{ij} of the Riemannian metric form a symmetric tensor of type (0, 2). [4]

c) If A_{ij} is a skew symmetric tensor, show that $\left(\delta^i_j\,\delta^k_\ell+\delta^i_\ell\,\delta^k_j\right)A_{ik}=0$. [2]

14. a) Show that the covariant derivative of a tensor of type (0,1) is a tensor of type (0,2) [5]

b) If A_i are the components of a covariant vector then show that $2\delta^i_j A_k+\begin{Bmatrix} i \\ jk \end{Bmatrix}$ are not components of a tensor. [5]

Unit – IV

[*Answer* ***any two*** *questions*]

15. a) Let τ, τ_u, τ_ℓ denote respectively the usual topology, the upper limit topology and the lower limit topology on $\mathbb{R}$. Compare the topologies τ, τ_u and τ_ℓ. What is the smallest topology on $\mathbb{R}$ containing both τ_u and τ_ℓ? Justify your answer. [4]

b) Prove that $\mathbb{R}$ with lower limit topology (i.e the sorgenfrey line) is not 2^{nd} countable. [3]

c) Prove that every closed subset of a Lindelöf space is Lindelöf. [3]

16. a) Prove that $\mathbb{R}$ with cocountable topology is not 1^{st} countable. [4]

b) State and prove pasting lemma. [4]

c) Give example of an open continuous map which is not closed. Justification needed. [2]

17. a) Prove that any two open intervals in $\mathbb{R}$ are homeomorphic. [3]

b) Let X and Y be two topological spaces where X is 1^{st} countable and let $f : X \to Y$ be a map. Prove that f is continuous if for every sequence $\{x_n\}$ in X converging to a point $x \in X$, the sequence $\{f(x_n)\}$ converges to f(x) in Y. [3]

c) Let (x, τ) and (y, τ') be two topological spaces and $f : (x, \tau) \to (y, \tau')$ be a function. Prove that f is continuous if and only if $f(\bar{A}) \subset \overline{f(A)}$ for every subset A of X. [4]

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